Techior Solutions Pvt. Ltd.

CBSE XII Maths Sample Paper

Total Time: 3 Hr Total Marks: 80.0

Maths

Section A

MCQ Single Correct

1) The area bounded by the parabola $y^2 = 4ax$ and the latus rectum is: **1.0**

A) $\frac{3}{8}a^2$

B) $\frac{8}{3}a^2$

C) $\frac{16}{3}a^2$

D) None of these

2) $\int \frac{dx}{e^x + e^{-x}}$ is equal to: **1.0**

A) $\tan^{-1}(e^{-x}) + C$

B) $\tan^{1}(e^{x}) + C$

C) $\log(e^x - e^{-x}) + C$

D) $\log(e^x + e^{-x}) + C$

3) Domain of the function $\sin^{-1}\left(\frac{2x+1}{3}\right)$ is: **1.0**

A) $(-2, 1)$

B) $[-2, 1]$

C) $(-2, 0)$

D) $[-1, 1]$

4) The general solution of the differential equation $e^x\,dy + (ye^x + 2x)\,dx = 0$ is **1.0**

A) $xe^y + x^2 = c$

B) $ye^y + x = c^2$

C) $ye^x + x^2 = c$

D) $ye^y + x^2 = c$

Techior Solutions Pvt. Ltd.

5) If $x, y \in R$ then the determinant $\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies in the interval **1.0**

A) $[-\sqrt{2}, \sqrt{2}]$
B) [-1, 1]
C) $[-\sqrt{2}, 1]$
D) $[-1, -\sqrt{2}]$

6) $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$ **1.0**

A) 0
B) π
C) $\frac{\pi}{2}$
D) none of these

7) Which of the following is a property of a symmetric matrix A? **1.0**

A) $A=A^T$
B) $A=-A^T$
C) $A=A^{-1}$
D) $A=-A^{-1}$

8) The area enclosed between the graph , $y = x^3$, and the lines x = 0, y = 1, y = 8 is: **1.0**

A) $\frac{45}{4}$
B) 14
C) 7
D) None of these

9) A triangle has a base of 8 cm and a height of 5 cm. What is the area of the triangle? **1.0**

A) 20 cm^2
B) 30 cm^2
C) 40 cm^2
D) 50 cm^2

10) If $\int x \sin x \, dx = -x \cos x + \alpha$, then α is equal to **1.0**

A) sin x + C
B) cos x + C
C) C
D) none of these

Techior Solutions Pvt. Ltd.

11) The function $f: R \rightarrow R$ defined by $f(x) = 2^x + 2^{|x|}$ is 1.0

A) one-one and onto
B) many-one and onto
C) one-one and into
D) many-one and into

12) If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is cofactors of a_{ij} then value of Δ is given by: 1.0

A) $A_{11} + a_{12}A_{12} + a_{13}A_{33}$
B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

13) The transportation problem in linear programming is a special case where: 1.0

A) The objective is to minimize transportation costs
B) The constraints are non-linear
C) There are no constraints
D) The feasible region is always bounded

14) The area bounded by the y-axis, y = cos x and y = sin x when $0 \le x \le \frac{\pi}{2}$ is 1.0

A) $1+\sqrt{2}$
B) $\sqrt{2}-1$
C) $\sqrt{2}+1$
D) $\sqrt{2}$

15) For real numbers x and y, define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is 1.0

A) reflexive
B) symmetric
C) transitive
D) none of these

16) Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line x + y = 2 is: 1.0

A) $\pi + 2$
B) $\pi - 2$
C) $2\pi - 1$
D) $2(\pi + 2)$

Techior Solutions Pvt. Ltd.

17) If x + y = k is normal to the curve $y^2 = 12x$ then k is equal to : **1.0**

- **A)** 3
- **B)** 9
- **C)** -9
- **D)** -3

18) If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then P (B | A) + P (A | B) equals **1.0**

- **A)** $\frac{1}{4}$
- **B)** $\frac{1}{3}$
- **C)** $\frac{5}{12}$
- **D)** $\frac{7}{2}$

<u>Section B</u>

Short Description

19) Find the intervals for which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is increasing or decreasing. **2.0**

---OR---

A particle moves along the curve $y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which y-coordinate changes as fast as x-coordinate.

20) Solve for x : **2.0**

$$\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2} = \frac{\pi}{3}$$

21) Find all points of discontinuity of f where **2.0**

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$$

---OR---

Examine the functions for continuity.

$f(x) = x - 5$

22) Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes **2.0**

$\vec{r} - (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$

23) Examine the consistency of the system of equation **2.0**

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & 2 & 6 \end{bmatrix},$$

Section C

Medium Description

24) If f (x), g (x) and h(x) are three polynomials of degree 2, then prove that **3.0**

$$\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix} \quad \phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$$ is a constant polynomial

---OR---

Differentiate the function w.r.t. x,

$$y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

25) Differentiate the function w.r.t. x, **3.0**

$$(\log x)^{\cos x}$$

---OR---

Differentiate the function w.r.t. x,

$$x^x - 2^{\sin x}$$

26) Integrate the function : $\frac{e^x}{(1+e^x)(2+e^x)}$ **3.0**

---OR---

Integrate the function :

$$\frac{1}{\sqrt{(2-x)^2+1}}$$

27) If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx} + y = 2\sqrt{x}$ **3.0**

28) Using properties of determinants prove that **3.0**

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

29) A kite is moving horizontally at the height of 151.5 meters. If the speed of kite is 10 m/sec. how fast is the string being let out: when the kite is 250 m away from the boy who is flying the kite? The height of the boy is 1.5 m. **3.0**

Tutor Solutions Pvt. Ltd.

Section D

Long Description

30) Prove that the image of the point $(3, -2, 1)$ in the plane $3x - y + 4z = 2$ lies on the plane x + y + z + 4 = 0. **5.0**

---OR---

Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through $(3, 0, -4)$. Also find the distance between these two lines.

31) If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ are the position vectors of the points A, B, C and D find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. **5.0**

---OR---

Using the vector method, find the values of λ and μ for which the points $A(3, \lambda, \mu)$, $B(2, 0, -3)$ and $C(1, -2, -5)$ are collinear.

32) The probability that at least one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P(\bar{A}) + P(\bar{B})$ **5.0**

33) Find the equation of the line passing through the point P (4, 6, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$ **5.0**

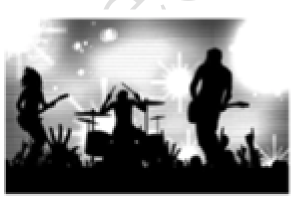
Section E

Case Study

Solve Question 34 to Question 38 based on the following paragraph:

Case Study 6:

Western music concert is organised every year in the stadium that can hold 36000 spectators. With ticket price of Rs. 10, the average attendance has been 24000. Some financial expert estimated that price of a ticket should be determined by the function p (x) = $15 - \frac{x}{3000}$, where x is the number of tickets sold.



Based on the above information, answer the following questions.

Excelsior Solutions Pvt. Ltd.

34) When the revenue is maximum, the price of the ticket is **1.0**

- **A)** Rs.5
- **B)** Rs.5.5
- **C)** Rs.7
- **D)** Rs. 7.5

35) The revenue, R as a function of x can be represented as **1.0**

- **A)** $15x - \frac{x^2}{3000}$
- **B)** $15 - \frac{x^2}{3000}$
- **C)** $15x - \frac{1}{30000}$
- **D)** $15x - \frac{x}{3000}$

36) The range of x is **1.0**

- **A)** [24000, 36000]
- **B)** [0, 24000]
- **C)** [0, 36000]
- **D)** none of these

37) The value of x for which revenue is maximum, is **1.0**

- **A)** 20000
- **B)** 21000
- **C)** 22500
- **D)** 25000

---OR---

How any spectators should be present to maximize the revenue?

- **A)** 21500
- **B)** 21000
- **C)** 22000
- **D)** 22500

38) How any spectators should be present to maximize the revenue? **1.0**

- **A)** 21500
- **B)** 21000
- **C)** 22000
- **D)** 22500

Solve Question 39 to Question 42 based on the following paragraph:

Case Study 7

Ajay cut two circular pieces of cardboard and placed one upon other as shown in figure. One of the circle represents the equation $(x - 1)^2 + 1 = 1$, while other circle represents the equation $x^2 + 1 = 1$.

Techior Solutions Pvt. Ltd.

Based on the above information, answer the following questions.

39) Value of $\int_0^{1/2}\sqrt{1-(x-1)^2}dx$ is **1.0**

A) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$

B) $\frac{\pi}{6}+\frac{\sqrt{3}}{8}$

C) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$

D) $\frac{\pi}{2}-\frac{\sqrt{3}}{4}$

40) Both the circular pieces of cardboard meet each other at **1.0**

A) x =1

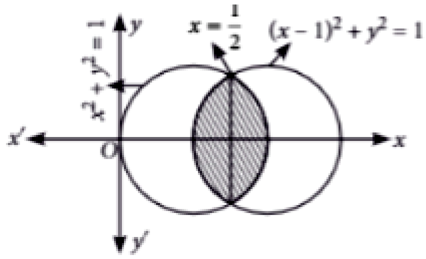
B) $x=\frac{1}{2}$

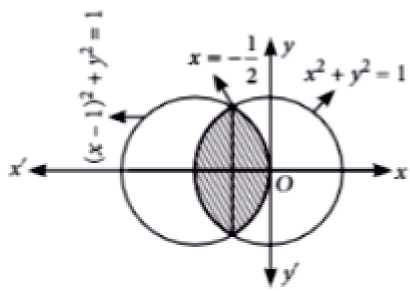
C) $x=\frac{1}{3}$

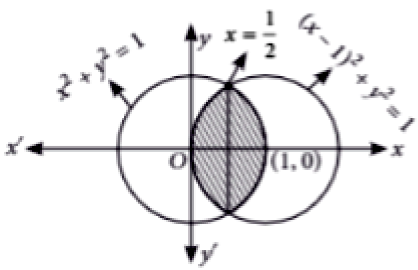
D) $x=\frac{1}{4}$

41) Graph of given two curves can be drawn as **1.0**

Techior Solutions Pvt. Ltd.

A) 

B) 

C) 

D) None of these

---OR---

Area of hidden portion of lower circle is

A) $\left(\frac{2\pi}{3}+\frac{\sqrt{3}}{2}\right)$ sq. units

B) $\left(\frac{\pi}{3}-\frac{\sqrt{3}}{8}\right)$ sq. units

C) $\left(\frac{\pi}{3}+\frac{\sqrt{3}}{8}\right)$sq. units

D) $\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$sq. units

42) Value of $\int_{1/2}^{1}\sqrt{1-x^2}dx$ is **1.0**

A) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$

B) $\frac{\pi}{6}+\frac{\sqrt{3}}{8}$

C) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$

D) $\frac{\pi}{2}-\frac{\sqrt{3}}{4}$

Solve Question 43 to Question 47 based on the following paragraph:

Case Study 5:

If an equation is of the form $\frac{dy}{dx} + Py = Q$ where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by $y \cdot (\text{I.F.}) = \int \text{Q} \cdot (\text{I.F.}) dx + c$ where $\text{I.F.} = e^{\int P dx}$

Now, suppose the given equation is

$$(1+\sin x)\frac{dy}{dx} + y\cos x + x = 0$$

Based on the above information, answer the following questions

43) Value of $y\left(\frac{\pi}{2}\right)$ is **1.0**

A) $\frac{4-\pi^2}{2}$

B) $\frac{8-\pi^2}{16}$

C) $\frac{8-\pi^2}{4}$

D) $\frac{4+\pi^2}{2}$

44) The value of P and Q respectively are **1.0**

A) $\frac{\sin x}{1+\cos x}, \frac{x}{1+\sin x}$

B) $\frac{\cos x}{1+\sin x}, \frac{-x}{1+\sin x}$

C) $\frac{-\cos x}{1+\sin x}, \frac{x}{1+\sin x}$

D) $\frac{\cos x}{1+\sin x}, \frac{x}{1+\sin x}$

45) The value of I.F. is **1.0**

A) 1 - sin x

B) cos x

C) 1 + sin x

D) 1- cosx

46) Solution of given equation is **1.0**

A) y{1-sinx)=x+c

B) y(l + sin x) = x^2+c

C) $y(1-\sin x)=\frac{-x^2}{2}+c$

D) $y(1+\sin x)=\frac{-x^2}{2}+c$

---OR---

If y(0) = 1, then y, equals

A) $\frac{2-x^2}{2(1+\sin x)}$

B) $\frac{2+x^2}{2(1+\sin x)}$

C) $\frac{2-x^2}{2(1-\sin x)}$

D) $\frac{2+x^2}{2(1-\sin x)}$

47) If y(0) = 1, then y, equals **1.0**

A) $\frac{2-x^2}{2(1+\sin x)}$

B) $\frac{2+x^2}{2(1+\sin x)}$

C) $\frac{2-x^2}{2(1-\sin x)}$

D) $\frac{2+x^2}{2(1-\sin x)}$

Techior Solutions Pvt. Ltd.